Helsinki University of Technology
CFD-group/ Laboratory of Applied Thermodynamics

MEMO No CFD/TERMO-14-97 DATE: January 13, 1997

TITLE

IMPLEMENTATION OF THE COMPRESSIBILITY EFFECT IN TO DISSIPATION EQUATION OF A TWO-EQUATION TURBULENCE MODEL

AUTHOR(S)
Patrik Rautaheimo

ABSTRACT

In the $k-\epsilon$ turbulence model the effect of compressibility is generally neglected and an incompressible version of the model is used. However, with hihg Mach numbers results can be improved by a compressible dissipation.

MAIN RESULT

Correction term owing to compressibility to the turbulent dissipation rate

PAGES
6

KEY WORDS
Dissipation, dilatation-dissipation, turbulent compressible flow

APPROVED BY

Timo Siikonen January 13, 1997

1 Introduction

Compressibility effects to the dissipation of the turbulent kinetic energy can be taken account through an extra dissipation term. This extra term has an effect to lower the turbulent kinetic energy rates in compressible flow. It also has an effect in heat transfer rates after a shock wave. A more general discussion can be found in [1].

2 Methods

Chien [2] $k-\epsilon$ turbulence model has been modified to take account dilatation-dissipation. According to Sarkar et al. [3] and Zeman [4],

$$\bar{\rho}\epsilon = \bar{\rho}(\epsilon_s + \epsilon_d) \tag{1}$$

where ϵ_s is a solenoidal dissipation and ϵ_d represents a compressible dissipation.

$$\epsilon_s = \bar{\nu}\overline{\omega_i^{\text{“}}\omega_i^{\text{“}}} \qquad \epsilon_d = \frac{4}{3}\bar{\nu}\overline{d^{\text{“}2}}. \tag{2}$$

Here $d^{''} = u^{''}_{k,k}$ denotes the fluctuating dilatation, $\omega_i^{\text{“}} = \epsilon_{ijk}u^{''}_{k,j}$ represent the fluctuating vorticity and ϵ_{ijk} is the permutation tensor. The solenoidal dissipation can be represented by the ordinary dissipation equation

$$\frac{\partial \rho\, \epsilon_s}{\partial t} + \frac{\partial \rho\, u_i\, \epsilon_s}{\partial x_i} = \frac{\partial}{\partial x_i}\left[(\mu + \mu_T/\sigma_\epsilon)\frac{\partial \epsilon_s}{\partial x_i}\right] + c_{\epsilon 1}\, f_1\, \frac{\epsilon_s}{k} P - c_{\epsilon 2}\, f_2\, \frac{\rho \epsilon_s^2}{k} + -2\mu(\epsilon_s/y_n^2)e^{-0.5y^+}. \tag{3}$$

Sarkar et al. [3] suggested for the compressible dissipation the following equation

$$\epsilon_d = Ma_t^2 \epsilon_s \tag{4}$$

where $Ma_t = \sqrt{\widetilde{u_i^{''}u_i^{''}}}/c$ is a turbulent Mach number and c is the speed of sound. Zeman model is more general [5]

$$\epsilon_d = \epsilon_s c_d \left\{1 - exp\left[-\left(\frac{M_{t^*} - M_{t_0}}{\sigma_M}\right)^2\right]\right\} \tag{5}$$

where $\epsilon_d = 0$ if $M_{t^*} \leq M_{t_0}$, with $M_{t_0} = 0.2$, $\sigma_M = 0.66$, $c_d = 0.75$ and

$$M_{t^*} = \sqrt{\frac{2}{\gamma + 1}} M_t \tag{6}$$

Because of Sarkar's model simplicity we are going to use only it.

3 Results

Compressible Flat Plate

As a first test case, flow over a flat plate is simulated. The case is with a compressible turbulence in zero-pressure-gradient boundary layer. Measurements where performed by Fernholtz et al. [6]. A similar calculation has been made by Ladeinde et al. [5]. The Reynods number per unit length, Re/m is $6.5 \times 10^7 1/m$ and $Ma_\infty = 2.87$.

The length of the domain is $0.6m$, the height $0.018m$ and the grid size is 192×64. The height of the first row of cells is $2 \times 10^{-6}m$. First three rows of the cells are kept constant and after that grid is clustered by ratio $\Delta y_{j+1}/\Delta y_j = 1.119$. Non-dimensional distance y^+ is close to one at the first row. At the leading edge y^+ is 3, and after 5 cells in axial direction, it is 1.

Inlet conditions are a uniform velocity distribution and pressure is extrapolated from the computational domain. Symmetry conditions are applied before the flat plate. At the flat plate, the velocities and kinetic energy of turbulence are set zero. The upper boundary parallel to the wall is assumed to be zero gradient. The second-order upwind scheme is used with Roe's flux splitting [7].

FINFLO INPUT file can be seen below

```
    1     1     3        0          |   IOLD    LEVEL     ITURB    NSCAL
/dionysos/tmp/rautahei/hilat/BOUNDARY_com.GRID
/dionysos/tmp/rautahei/boundary/Compressible/BOUND.BC
  'ROE'  'YES'  'NO'  'NO' 'YES'    |   IFLX   RESTART  STRESC  FULLFC SOURC
 'NO'    'YES'    0     0           |   TIMEC   CONVC   PRESC   LUSGS
  .5   .5  .00000001 64368. 1.0     |   CFL-NUMBERS   DROLIM  TMAX    DT1
   10000  10000    1000      1      |   ICMAX   KPRINT  MPRINT  IDRXX
   0.0   0.000    00.0    0.000     |   RMACH    ALPHA   BETA  ROTAT
    0.0      0.72    0.90           |   RE   PRANDTLIN LUVUT(PR;PRT)
   1            'YES'               |   ISTATE      STATEC
  96.3   0.8478  0.       564.0     |   FRSTEM  FRSDEN  FRSPRE  FRSVEL
   0.0002  .1   5000.     1         |   TURBLIM   RMULIM TURLIM  IESPMA
   0.0003  .2   1.   2.     5.      |   TURBFRS   RMUFRS    CMGK CMGEPS  CSIMPS
   0       0          0     0      |   JRDIF     JRDIS   JRPRE  JRIMP
  0.1       1.                     |   CC1    CC2
  .6e-2     1.       1.            |   AREF  CHLREF   GRILEN
  0.     0.    0.  0.  -1.         |   XMOM  YMOM   ZMOM  REFPRE  DIFPRE
  0.     0.    0.  0.              |   GROUND  GI      GJ      GK
   4    3      0                  |   NUMBER OF BLOCKS   NOF INLETS/OUTLETS
 192   64   01    +1 -3 -1  010  1 |   IMAX JMAX KMAX INTER(IJK) LAMIN BLOCK 1
  00   3     1   193  65    2    0 |   IT   IL   IK     IDI1   IDI2  IDI3 MOV
   2   1   192     3  64    1   01 |   MGRID  MIB MIT   MJB MJT   MKB MKT
  0.   0     0                     |   OMEGA IROTVE    IGRID
   2   64   01    +1 -1 -1  010  1 |   IMAX JMAX KMAX INTER(IJK) LAMIN BLOCK 1
   0   3     1     3  65    2    0 |   IT   IL   IK     IDI1   IDI2  IDI3 MOV
   1   1     2     3  64    1   01 |   MGRID  MIB MIT   MJB MJT   MKB MKT
  0.   0     0                     |   OMEGA IROTVE    IGRID
 192   2    01    +1 -1 -1  010  1 |   IMAX JMAX KMAX INTER(IJK) LAMIN BLOCK 1
  0    3     1    97   3    2    0 |   IT   IL   IK     IDI1   IDI2  IDI3 MOV
  1    1    96     1   2    1   01 |   MGRID  MIB MIT   MJB MJT   MKB MKT
  0.   0     0                     |   OMEGA IROTVE    IGRID
   2   64   01    +1 -1 -1  010  1 |   IMAX JMAX KMAX INTER(IJK) LAMIN BLOCK 1
   0   3     1     3  65    2    0 |   IT   IL   IK     IDI1   IDI2  IDI3 MOV
   1   1     2     3  64    1   01 |   MGRID  MIB MIT   MJB MJT   MKB MKT
  0.   0     0                     |   OMEGA IROTVE    IGRID
```

Velocity profiles are compared at $x = 0.5m$ with the experimental result

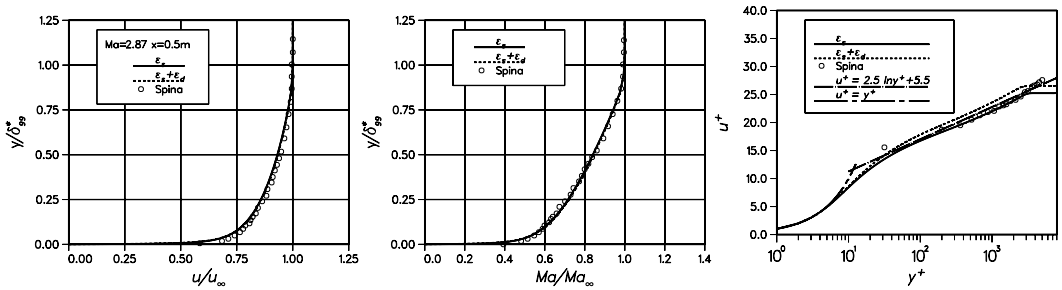

Fig. 1: Comparision of the velocity profiles with the measurements of Fernholtz et al. [6].

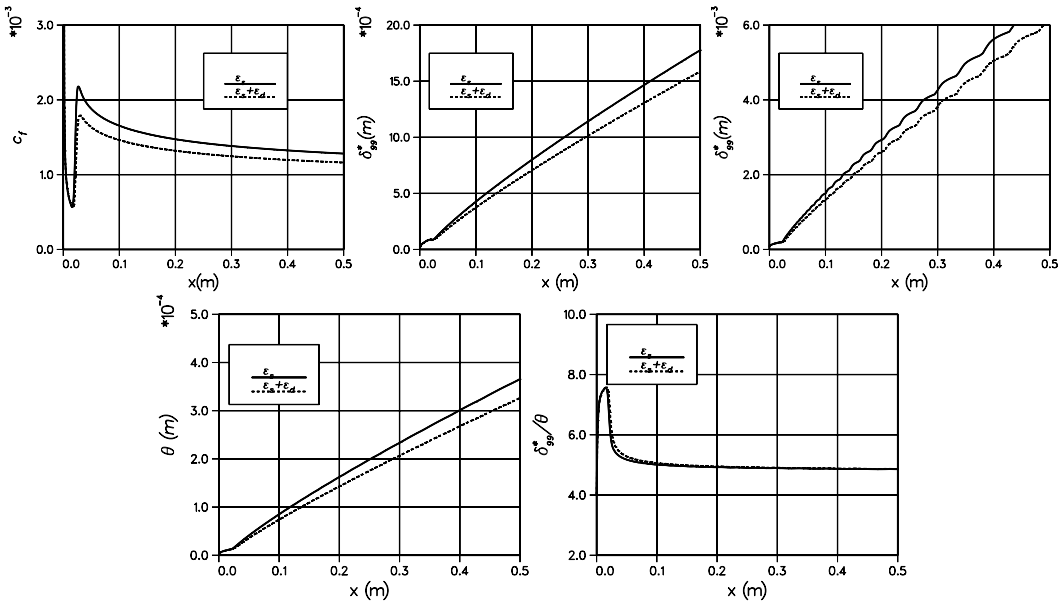

Fig. 2: Universal boundary layer parameters. From left to right: friction coefficient, boundary layer thickness, displacement thickness, momentum thickness and shape function.

in Fig. 1. Velocity profiles show good agreement with experiments. At the universal velocity profiles there is small variation between methods and experiments. Universal boundary layer parameters are shown in Fig. 2. Boundary layer parameters are in figure from left to right: friction coefficient c_f, boundary layer thickness at point where velocity is 99% of the free stream velocity δ_{99}, displacement thickness δ^*_{99}, momentum thickness θ and shape function $H = \delta^*_{99}/\theta$. These variables are defined as

$$c_f = \tau_w / \left(\frac{1}{2}\rho U_e^2\right) \tag{7}$$

$$\delta^* = \int_0^{\delta_{99}} (1 - \frac{u(y)}{U_e}) dy \tag{8}$$

$$\theta = \int_0^{\delta_{99}} \frac{u(y)}{U_e}(1 - \frac{u(y)}{U_e}) dy \tag{9}$$

where U_e velocity at the edge of the boundary layer. No experimental results were available. As it can be seen in Fig. 2 spreading rate and skin friction are smaller with compressible dissipation.

NACA 0012

As a second test case, flow over a NACA 0012 airfoil is calculated at $Ma = 0.799$, $Re = 9 \times 10^6$ and $\alpha = 2.26^o$. A C-type grid with 192×64 cells is used in the simulation. The outer boundary of the grid is 20 chord lengths from the airfoil, and the cell thicknesses on the surface varies from 5×10^{-6} at the leading edge to 2×10^{-5} at the trailing edge. Nondimensional distance y^+ is from 2 to 5. Turbulence model is Chien [2] $k-\epsilon$ with Sarkar's modification. No transition model is used and Chien's model simulates the transition by itself. In the simulation, transition take place just at the leading edge for both size. The case has been previously calculated by Siikonen [8]. Experimental results for the force coefficient and the pressure coefficient distribution is from Holst [9].

FINFLO INPUT file can be seen below

```
 NACA 0012
   1      1       3     0           |   IOLD  LEVEL   ITURB  NSCAL
/dionysos/tmp/rautahei/hilat/NACA2D.GRID
/dionysos/tmp/rautahei/naca2d/NACA.BC
  'ROE'   'YES'   'NO'   'NO'   'NO'    |   IFLX    RESTART STRESC  FULLFC  SOURC
  'NO'    'YES'    0      0            |   TIMEC   CONVC   PRESC   LUSGS
  2.5      2.5  1.E-7 64368. 1.0       |   CFL-NUMBERS     DROLIM  TMAX    DT1
  5000    10000    1000      1         |   ICMAX   KPRINT  MPRINT  IDRXX
 0.799    2.26    00.0    0.0          |   RMACH   ALPHA   BETA    ROTAT
 9.E6     0.72    0.90                 |   RE      PRANDTLIN LUVUT(PR;PRT)
   1      'YES'                        |   ISTATE  STATEC
 288.15     0.     0.     0.           |   FRSTEM  FRSDEN  FRSPRE  FRSVEL
  2.E-4   .1458   5000.    1           |   RKLIM   MULIM   TURLIM  IESPMA
  0.001    1.0    0.10     0.2   5.    |   TURBINI RMUINI  CMGK    CMGEPS  CSIMPS
   0        0      0        0          |   JRDIF   JRDIS   JRPRE   JRIMP
  0.1       1.                        |   CC1     CC2
  0.01      1.     1.                 |   AREF    CHLREF  GRILEN
  0.        0.     0.       0.   -1.   |   XMOM    YMOM    ZMOM    REFPRE  DIFPRE
  0.        0.     0.       0.         |   GROUND  GI      GJ      GK
  1         0      0                  |   NUMBER OF BLOCKS   NOF INLETS/OUTLETS
 192  64   01     +1 -3 -1  010   1   |   IMAX  JMAX KMAX INTER(IJK) LAMIN BL 1
 161   3    1    193    65    2   0   |   IT    IL   IK   IDI1    IDI2  IDI3 MOV
  3    1  192      2    64    1  01   |   MGRID MIB  MIT  MJB MJT MKB MKT
  0.   0    0                         |   OMEGA IROTVE    IGIRD
```

This case is calculated with and without Sarkar modification. Iteration history for both cases can be seen in Figure 3. Stability of both methods are similar. The force coefficients can be seen in Table 1. It can be seen that Sarkar's modification does just only slightly better than the ordinary dissipation equations. However, qualitatively the result is slightly better with Sarkar model. This can be seen in Figure 4. Sarkar's model simulates even a boundary layer separation, but separated region is far too short. This is a common phenomenon with $k-\epsilon$ turbulence models. Also overprediction of the skin friction after a shock can be seen with both models.

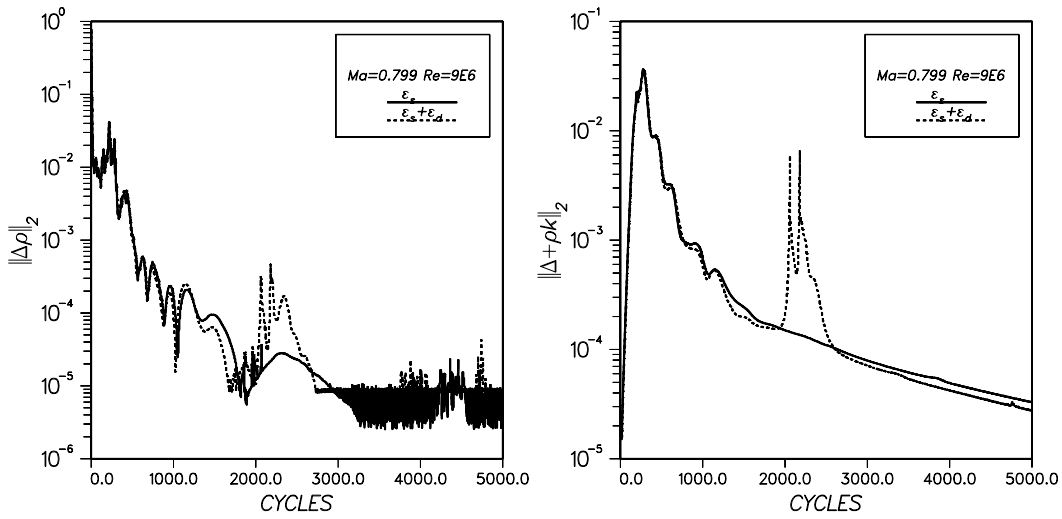

Fig. 3: Convergence history of the simulation.

Table. 1: Lift and drag coefficient.

	c_l	c_d	c_l/c_d
Experimental	0.390	0.0331	11.78
Normal dissipation	0.332	0.0391	8.49
Sarkar model	0.330	0.0382	8.64

4 Discussion

In these test cases the compressible dissipation did not have a big effect. It makes results a bit better. It reduces skin friction after a shock wave and also reduces the spreading rate of the boundary layer. The compressibility effects could be more visible with highly supersonic as hypersonic flows.

References

[1] Ladeinde, F., "Supersonic Flux-Split Procedure for Second Moments of Turbulence," *AIAA Journal*, Vol. 33, July 1995, pp. 1185–1195.

[2] Chien, K.-Y., "Predictions of Channel and Boundary-Layer Flows with a Low-Reynods-Number Turbulence Model," *AIAA Journal*, Vol. 20, Jan 1982, pp. 33–38.

[3] Sarkar, S., Erlebacher, B., Hussaini, M., and Kreiss, H., "The Analysis and Modelling of Dilational Terms in Compressible Turbulence," *Journal of Fluid Mechanics*, Vol. 227, 1991, pp. 473–493.

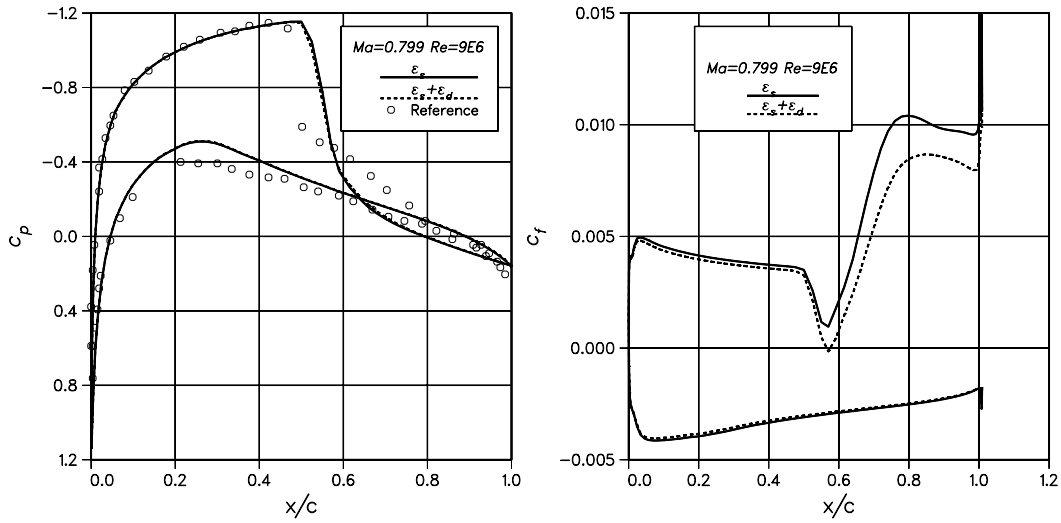

Fig. 4: Pressure and friction coefficient distributions.

[4] Zeman, O., "Dilation dissipation: the Concept and Application in Modeling Compressible Mixing Layers," *Phys. Fluids A*, Vol. 2, 1990, pp. 178–188.

[5] Ladeinde, F. and Intile, J., "Calculation of Reynolds Stresses in Turbulent Supersonic Flows," *International Journal for Numerical Methods in Fluids*, Vol. 21, 1995, pp. 49–74.

[6] Fernholtz, H., Finley, P., Dussauge, J., and Smits, A., "A Survey of Measurements and Measuring Techniques in Rapidly Distorted Compressibble Turbulent Boundary Layers," AGARD 315, 1989.

[7] Roe, P., "Approximate Riemann Solvers, Parameter Vectors, and Difference Schemes," *Journal of Computational Physics*, Vol. 43, 1981, pp. 357–372.

[8] Siikonen, T., "An Application of Roe's Flux Difference Splitting for the $k-\epsilon$ Turbulence Model," Helsinki University of Technology, Laboratory of Aerodynamics, 1994. ISBN 951–22–2059–8.

[9] Holst, T., "Viscous Transonic Airfoil Workshop Compendium of Results," *Journal of Aircraft*, Vol. 25, Dec 1988, pp. 1073–1087.